REVISTA ESPACIOS

Management and modeling of the influence of cracks on the temperature distribution in polymers and composites

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Abstract
When exposed to a steady heat flow in a sample with a crack is an increase in thermal stress caused by a local increase of the temperature gradient in the vicinity of the crack. The presence of cracks distorts the temperature field near cracks, the dimensions of the distortion are determined by the size of the crack. On the crack in addition to the displacement jump occurs, the jump in temperature proportional to the power of the external heat flux and size of the crack. In the mechanical field, a crack is a hub (local amplifier) voltage, and temperature field, in addition to the hub of the heat flow. There was produced a detailed analysis of the temperature field in the specimen with a crack.
Key words
crack, temperature field

1. Introduction

Fracture of solids, in particular polymers and composites on their basis is a process of accumulation of internal microdamages to a certain critical point (Dimitrienko and Sokolov, 2013; 2012; 2010; Dmitroenko, 1995; 1997). This process is localized predominantly at the weakest points of the structure of the material, where there are pockets of surge in which mechanical stress is much more than away from them. Such foci are, first, micro and macro.

The model and the theory of fracture of polymers and composites on their basis under non-isothermal conditions is the least developed region in the science of strength of solids. There are practically important cases when it is impossible to consider the temperature field of the sample uniform, and to consider non-uniform temperature distribution and, in particular, the perturbation of the temperature field caused by the presence of cracks. You can consider a stationary temperature field, as transient non-stationary processes, as a rule, quickly enough damped.

Experimental data (Finkel, 1977) indicate that in the case of steady heat flow in a sample with a crack is a significant increase in thermal stress caused by a local increase in the magnitude of the temperature gradient in the vicinity of the crack. In this case thermoelastic compressive stresses being imposed on the elastic field of mechanical stresses that can reduce the total stress intensity at the crack tip and thereby can slow down its development.

In turn, the thermoelastic tensile stress can cause crack growth and damage to the specimen even in the absence of mechanical stress. Experiments confirm this conclusion. In the monograph [6] described the experiment, when the plate of a polymeric material with an internal crack, would be destroyed only under the influence of the external temperature field, without mechanical action. Comparison of different experimental results concerning the observations of Cremieu in terms of mechanical or thermal loading of the sample, allowed us to formulate the most important General conclusion: the crack grows under the action of local stresses in the apex. It doesn't matter what factor created by these local stresses: external mechanical stress, inhomogeneous temperature field or maybe even some other factor.

The singular character of thermal stress near the tip of the crack is analyzed in (Si, 1963) where it is shown that the classical conception about the peculiarities of the mechanical stress near the tip of the crack remain in force and thermoelastic stresses. The presence of heat flow in the specimen with a crack does not cause additional singularities, so singulares of thermal stress is the usual form , $\frac{K}{\sqrt{r}}$ where *r* is the distance from the top of the crack, and *K* the ratio of the intensity of thermal stress, which is calculated in each case of thermal loading (Kartashov, 1991). The difference fields of thermoelastic stresses from the mechanical stress is the intensity factor *K*, which has a different kind in these two cases.

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2. Analysis of the temperature field in the specimen with a crack

We formulate a thermal problem corresponding to the above-mentioned experiment, described in [6]. Consider a sample in the form of thin strips (or plates) of length 2b and width 2a with an internal, end-to-end crack length *2l* (Fig. 1).

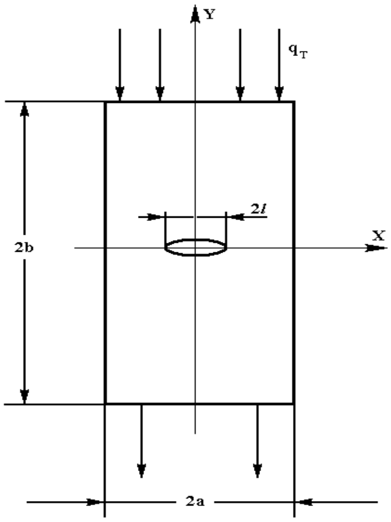

Fig. 1 To the problem of temperature field of the specimen with a crack

Along the y-axis runs parallel to the sample plane steady-state heat flow is constant power q_T . The crack is a cavity in the sample, and the shore opened cracks separated by a layer of air (or even vacuum) with thermal conductivity much lower than that of the sample material. Therefore, the heat transfer through the crack is virtually absent, and can be regarded as impermeable to heat flow.

In the beginning of the process, a starting flow of heat in the sample occurs non-stationary temperature distribution. Over time transients are damped, and installed stationary distribution that does not depend on time. This stationary state will be considered.

Analysis of experimental data shows (Regel et al., 1974) that the internal microcracks in polymers and composites on their basis have the initial sizes of the order of 0.03-0.3 µm, predestiny critical size of the crack is about six to ten times more, i.e. also very small. As an example, consider the sample strip PMMA length 2b=20 mm and width 2A=5 mm (this is the typical dimensions of samples used in the durability test). The initial crack in PMMA has a size of order l_0 =0,23 µm [9] . For such a sample $x=\frac{a}{b}=0.25$ Critical prekatrina the crack length can reach six to ten initial sizes, i.e. $l_{кр}\sim 1{,}4\div 2{,}3$ microns. Then $\frac{l_{кр}}{2a}\approx(2.8-4.6)\cdot 10^{-4}$. In any case, the size of crack is small compared with specimen dimensions. The distortion of the temperature field caused by such a crack is concentrated in a small area. All this allows in accordance with the known principle of a microscope is taken as the mathematical model of the problem of elastic plane with a slit along a cut $|x|\le l,\ \ y=0$ simulating a crack.

In this formulation, the problem about the stationary temperature distribution in the specimen with a crack has the form:

$$\begin{gathered}\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0\\ (x,y)\in R^2\setminus |x|<l, y=0\\ \frac{\partial T}{\partial y}\Big|_{y\to\pm\infty}=\frac{q_T}{\lambda_T}\\ \frac{\partial T}{\partial y}\Big|_{y=0,|x|<l}=0\end{gathered} \qquad (2)$$

Approximation of sample strips of finite size with a small crack in an infinite plane is physically justified and mathematically, the problem becomes solvable.
Problem (2) belongs to the class of internal problems Neumann. Its solution is determined up to an additive constant and is equal to (Kartashov, 1985).

$$T(x,y)=T_0+\frac{q_T}{\lambda_T\sqrt{2}}\left[\sqrt{y^4+2y^2\left(x^2+y^2\right)+\left(x^2-l^2\right)^2}+y^2-\left(x^2-l^2\right)\right]^{\frac{1}{2}} sign\, y \qquad (3)$$

This formula determines the temperature field in the "large sample with a small crack." The constant T_0 cannot be determined from the conditions of the boundary value problem (2). In order to understand its meaning, make a limit transition in the formula (3), $l \to 0$ provided, i.e. the transition to the specimen without the crack. The temperature distribution in this sample is equal to

$$T(x,y)=T_0+\frac{q_T}{\lambda_T}y \qquad (4)$$

The same result is obtained by solving the transient boundary value problem for a finite sample without cracks with the initial temperature T_0 and the subsequent limiting transition to the steady state. In this case, the constant T_0 is equal to the initial temperature of the sample before the heating process. Therefore, it can be argued that in the formula (3) constant T_0 has the same meaning, and therefore the second term in this formula describes the deviation of temperature field from the initial state. For large values of x and y, i.e. away from the crack, the temperature field remains undistorted and is described by the formula (4).
On the line of the crack (y = 0) temperature is

$$T(x,0)=\begin{cases} T_0\pm\frac{q_T}{\lambda_T}\sqrt{l^2-x^2} & ,\ |x|<l \\ T_0 & ,\ |x|>l \end{cases} \qquad (5)$$

and the plus sign refers to the upper part of the sample (above the crack), and the minus sign – to the lower part (under the crack). Figure 2 shows the temperature profile on the crack. The curve in this figure is the ellipse with equation

$$\frac{\left(T(x,0)-T_0\right)^2}{\left(\frac{q_T}{\lambda_T}\right)^2 l^2}+\frac{x^2}{l^2}=1 \qquad (6)$$

The vertical semiaxis of the ellipse determines the maximum temperature change on the crack faces (heating on the upper shore and the cooling lower), it is achieved in the middle of the crack and is equal to

$$\Delta T_{\max}=\frac{q_T}{\lambda_T}l \qquad (7)$$

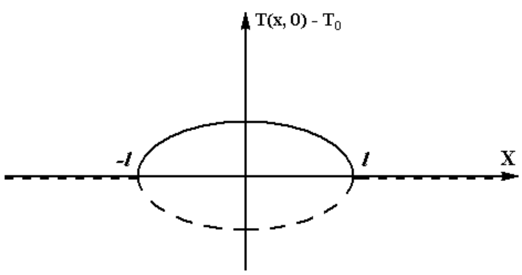

Fig. 2. The temperature profile on the line of the crack. The solid line is the upper Bank of the fissure and dotted the lower shore.

The closer to the top of the crack growth $x=\pm l$ temperature decreases ΔT, turning to zero at the ends, and outside of the crack growth temperature is zero, i.e. where it is stored initial temperature. Thus, on the line of the crack out of it all $(y=0, |x|>l)$ the time supported the initial temperature T_0. Area of the temperature change in horizontal is limited by the size of the crack.

Rate the length of the zone of the distortion of the temperature field of a crack in the vertical direction as the distance from the crack. This is more convenient to switch to dimensionless variables, measuring the spatial distances on x and y axes are in units that are multiples of the crack length $l: \mu=\frac{x}{l}$, $\varepsilon=\frac{y}{l}$. In these variables the equation (3) takes the form:

$$T(\mu,\varepsilon)=T_0+\frac{q_T l}{\lambda_T}\left[\frac{1}{2}\left(\sqrt{\varepsilon^4+2\varepsilon^2\left(\mu^2+1\right)+\left(\mu^2-1\right)^2}+\varepsilon^2-\mu^2+1\right)\right]^{\frac{1}{2}} sign\,\varepsilon \qquad (8)$$

This formula was designed equidistant temperature field (Fig.3), i.e. the line along which constant dimensionless quantity.

$$\Delta(\mu,\varepsilon)=\frac{T(\mu,\varepsilon)-T_0}{\frac{q_T}{\lambda_T}l} \qquad (9)$$

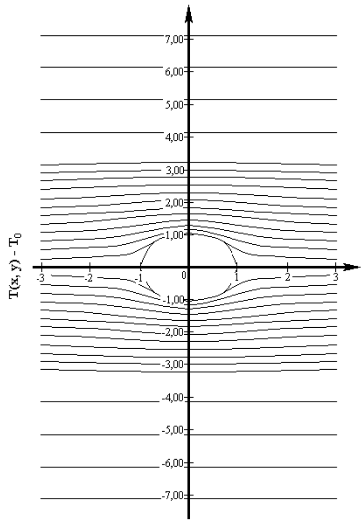

Figure 3. Equidistante the temperature field near cracks

Each offset in this figure corresponds to the vertical distance from the crack, expressed in units of multiples of the size of the crack $l\left(\varepsilon=\frac{y}{l}\right)$. Pattern in Fig.3 shows the distribution of relative temperature in dimensionless units across the width of the sample at different distances vertically from the cracks. In the sample without cracks equidistantly represent straight lines parallel to the x-axis. The presence of cracks distorts the picture. From Fig.3 shows that as the distance from the cracked distortion of the temperature field decreases, and, in the end, equidistant converted into the horizontal parallel lines. The degree of distortion of the temperature field is characterized by the difference

$$\delta(\mu,\varepsilon)=\Delta(\mu,\varepsilon)-\varepsilon \qquad (9a)$$

Function in regardless $\delta(\mu,\varepsilon)\rightarrow 0$ when $\varepsilon\rightarrow\infty$ regardless μ. In Fig. 4 shows the relative distortion of the temperature field $\frac{\delta(\mu,\varepsilon)}{\varepsilon}$ in comparison with the sample without cracks the same distances from the crack vertically.

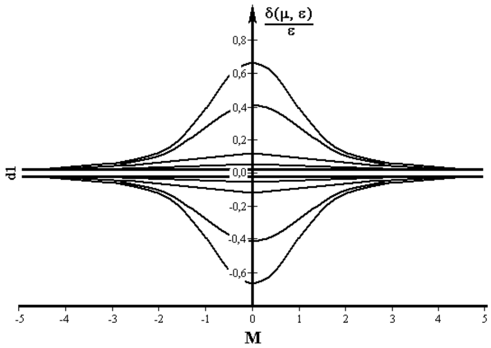

Fig.4 Relative distortion of the temperature field caused by the crack

In Fig. 5 shows the maximum relative distortion of the temperature field at different distances from it.

$$\frac{\delta(0,\varepsilon)}{\varepsilon}=\sqrt{1+\frac{1}{\varepsilon^2}}-1 \quad (10)$$

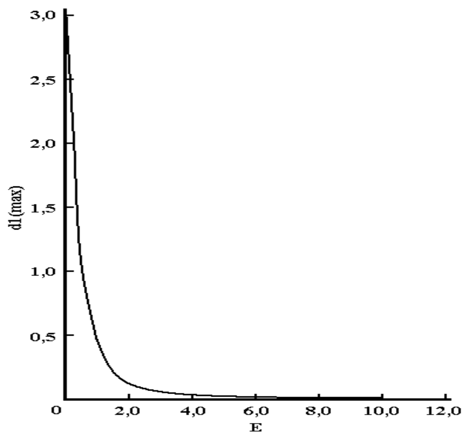

Fig. 5. The maximum relative distortion of the temperature field near the top of the crack at different distances from it

It is seen that it decreases rapidly with distance from the crack. In particular, at distances of $\varepsilon \approx 10l$ maximum distortion is 0.5%, and at a distance $\varepsilon \approx 30l$ - already 0,06%. Summing up, it can be argued that the zone of perturbation of the temperature field extends to a distance $10-20l$ from the crack up and down. For example, for PMMA the relative length of the zone of disturbance $0.02-0.1\%$ is the longitudinal size of the sample, i.e. covers a small part of it.

We now investigate the distribution of heat flow in the specimen with a crack. For this purpose, find the vector of heat flux density

$$q=-\lambda_T \, grad\, T=-\lambda_T\left(\frac{\partial T}{\partial x}i+\frac{\partial T}{\partial y}j\right)=q_x i+q_y j \quad (11)$$

Differentiating the formula (3), we obtain

$$
\begin{aligned}
&а)\ q_x = -\frac{q_T}{\sqrt{2}} B^{-\frac{1}{2}} x\left(\left(r^2 - l^2\right) A^{-1} - 1\right) sign\, y \\
&б)\ q_y = -\frac{q_T}{\sqrt{2}} B^{-\frac{1}{2}} y\left(\left(r^2 + l^2\right) A^{-1} + 1\right) sign\, y \\
&в)\ r^2 = x^2 + y^2 \\
&г)\ B(x,y) = A(x,y) + y^2 - \left(x^2 - l^2\right) \\
&д)\ A(x,y) = \sqrt{y^4 + 2y^2\left(x^2 + l^2\right) + \left(x^2 - l^2\right)}
\end{aligned}
\qquad (12)
$$

We investigate the asymptotic behavior of the vector of heat flux density q away from the crack (for large y and x). For large y, regardless of x have

$$\lim_{|y|\to\infty} q_x(x,y) = 0, \quad \lim_{|y|\to\infty} q_y(x,y) = -q_T \qquad (13)$$

For large x, irrespective of y will be

$$\lim_{|x|\to\infty} q_x(x,y) = 0, \quad \lim_{|x|\to\infty} q_y(x,y) = -q_T \qquad (14)$$

i.e. away from the crack vector of heat flux density equal

$$q(x,y) = -q_T\, j \qquad (15)$$

ie has the same form as in the sample without cracks. This again confirms the earlier conclusion that the distance from the crack the temperature field remains undisturbed.

On the line of the crack at y=0 the longitudinal and transverse components of the vector q equal to

$$
\begin{aligned}
&а)\ \lim_{y\to\pm 0} q_x(x,y) = \begin{cases} \pm q_T \dfrac{x}{\sqrt{l^2 - x^2}}\ , & |x| < l \\ 0\ , & |x| > l \end{cases} \\
&б)\ \lim_{y\to\pm 0} q_y(x,y) = \begin{cases} 0\ , & |x| l \\ -q_T \dfrac{|x|}{\sqrt{x^2 - l^2}}\ , & |x| > l \end{cases}
\end{aligned}
\qquad (16)
$$

The plus sign here refers to the upper end of the crack, the minus sign the lower shore.

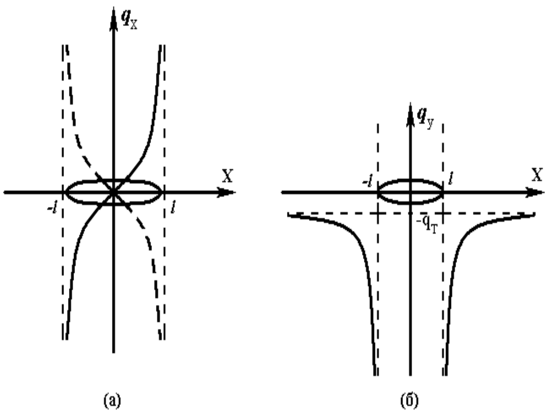

Fig.6. The distribution of tangential (a) and normal (b) components of the vector of heat flux density across the width of the sample in the vicinity of the crack-cut. a) tangential component. The solid line is the upper Bank of the fissure, the dashed line represents the lower shore. b) the normal component of the upper shore. On the lower shore, the picture of the exact same.

In Fig. 6 shows a diagram based on component q_x and q_y the vector of heat flux density on the coordinate x on the upper and lower crack faces. On the upper shore, the tangential component q_x increases in absolute terms indefinitely approaching the heights of the crack. Out cracks the tangential component q_x is everywhere equal to zero. The normal component, q_y on the contrary, inside the crack is missing, and out of the cracks as the distance from the peaks rapidly decreasing in absolute value from an infinite value to a level corresponding to the absence of cracks.

The appearance of infinities for the components of the vector of heat flux density q is a flaw in the accepted model of the crack. In reality, the crack, of course, is not a mathematical section, and not having cracks. The most advanced model of brittle crack Rehbinder – Bartenev – Razumovskaya (RBR - model) (Bartenev, 1984) considering the crack as the crack with asymptotically converging shores. Therefore, physically real picture of the distribution of heat flux such as shown in Fig. 7.

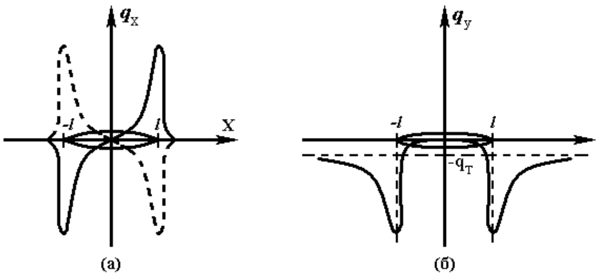

Fig. 7. The distribution of tangential (a) and normal (b) components of the vector of heat flux density across the width of the sample in the area of real cracks. The designations are the same as in Fig. 6

The tangential component $q_x$approximation to the vertices of the crack inside the crack rather slowly increases in absolute value to large, but finite values, and out cracks very quickly decreases to zero (Fig. (7a). For the normal components q_y of the picture will be the same as in Fig. 7b, i.e. away from the crack the normal component slowly decreases in absolute value to the value q_T corresponding to the condition with no cracks and inside the cracks with the approach to its peaks rising sharply in modulus from zero to a large value. On the lower banks of the crack the normal component q_y behaves the same as on the top.

The physical interpretation of the picture in Fig. 7 is as follows. The crack is impermeable to heat flow. This means that the normal component of the vector of heat flux density q_y on the upper end of the crack is zero and the tangential component $q_x$is non-zero and increases a s it approaches the vertex of the crack from the inside. This means that the heat "flows" along the upper Bank of the fissure, diverge to the left and to the right of its middle. At the same time through each unit area, perpendicular to the line of the crack, i.e. the x-axis, is moved an increasing amount of heat, and through the same site, located in the top of the crack is transferred the greatest amount of heat. This is because along the top of Bank "flows," the heat that falls "on top of" an impenetrable shore of cracks and "wrapping" flowing along the shore. Therefore, the farther from the middle of the crack, the greater the amount of heat falling on a crack and "wraps" along it. Close to the top of the partially cracked heat "leaks" through the Bank, because here is the stretched, weakened, but not broken interparticle connection, through which occurs the leakage of heat.

Let us now consider the situation outside of the crack from its upper Bank. Away from the vertices of the crack normal vector component of heat flux density, as in no cracks, has a negative sign. This means that the heat "flows" from top to bottom (Fig. 7b) . As you get closer to the heights of the cracks on the outside of the normal component q_y increases in absolute value. This means that in a plane perpendicular to the y-axis (i.e. a horizontal platform on the line of the crack), transferred an increasing amount of heat as you get closer to the top of the crack. The reason is that using such a site is transferred, firstly, the "normal" amount of heat corresponding to the absence of cracks, and, secondly, incremental heat, which "flows" along the crack and around the top. The closer to the crack tip, the larger is the added amount of heat. In the top transferred the largest amount of heat in the vertical direction. "Inside" cracks curve q_y sharply decreases to zero, which corresponds to the tightness of the crack. From Fig. 7 and formulas (15), shows that the height of peaks in the tops of cracks are the same, i.e. the amount of heat that "surrounds" the top of the crack equals the amount of heat passing vertically downwards.

Let us now consider the situation on the lower Bank of the fissure. The behavior of the tangential components q_x of the vector of heat flux density is shown in figure 7a as a dashed line. The behavior of the normal components below the line of the crack is the same as for the upper shore. In Fig. 7a it is seen that inside the crack there is a non-zero horizontal heat flux along the lower shore, and the maximum value of flow observed at the vertices of cracks. This means that the heat that goes around the top of the cracked "bleeding" under the crack. Ie there is the phenomenon of a kind of "heat diffraction" and the lack of "heat shadows" through the cracks. As you get closer to the middle of the crack horizontal heat flux coming from the vertices along the bottom of the shore decreases, i.e. the amount of heat transported perpendicular to the line of the crack decreases as it approaches the middle of the crack. This is because the flow of heat along the lower shore, gradually turns down, ie, at the slightest deviation from the lower shore appears normal component q_y, which is greater, the closer to the middle of the crack.

In the end, the distribution pattern of vector lines in the heat flux q appears as shown in figure 8. Heat "flows" as if in the heat pipe around the obstacle is a crack.

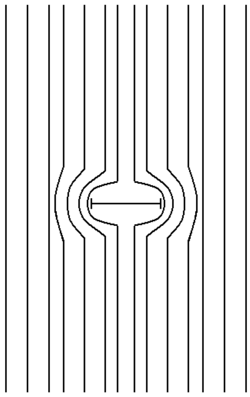

Fig. 8. the Pattern of vector lines of the temperature field in the specimen with an internal crack

In the intact sample without cracks temperature field T(x,y) continuously at each point of the sample. The presence of cracks causes a leap of the temperature inside the cracks due to the impermeability of its shores. On the upper end of the crack and close it creates a zone of elevated temperature. From the formula (8), we obtain in dimensionless coordinates near the upper shore

$$T(\mu,\varepsilon)=T_0+\frac{q_T}{\lambda_T}l\sqrt{1-\mu^2}\left(1+\frac{2+\mu^2}{4\left(1-\mu^2\right)}\varepsilon^2\right) \quad (17)$$

Most of the temperature increase achieved in the mid-upper shore $\mu=0$

$$T(0,\varepsilon)=T_0+\frac{q_T}{\lambda_T}l\left(1+\frac{1}{2}\varepsilon^2\right) \quad (18)$$

Below the crack is the zone of low temperature, where the temperature is lower than the initial one T_0. It is a zone of partial "heat shadows", the temperature is equal

$$T(\mu,\varepsilon)=T_0-\frac{q_T}{\lambda_T}l\sqrt{1-\mu^2}\left(1+\frac{2+\mu^2}{4\left(1-\mu^2\right)}\varepsilon^2\right) \qquad (19)$$

The temperature jump on the crack faces equal

$$\Delta T(\mu)=\frac{2q_T}{\lambda_T}l\sqrt{1-\mu^2} \qquad (20)$$

Its greatest value is

$$\Delta T_{\max}=\frac{2q_T}{\lambda_T}l \qquad (21)$$

The closer to the top of the crack the temperature jump tends to zero. The magnitude of the temperature jump on the crack is determined by the capacity of influencing the heat flux q_T and the crack length l. Numerical estimation for PMMA yields a value $\Delta T_{\max}$ of from tenths of a degree to several degrees, but the closer to the top of the crack the temperature of its coast lines.

We consider in detail the temperature distribution and heat flow in proximity to the vertices of the crack. To do this, switch to the polar coordinate system by placing its beginning at the right crack tip (Fig 2). In the vicinity of the left vertex the situation is similar. Temperature distribution in polar coordinates has the form:

$$T(\xi,\theta)=T_0+\frac{q_T}{\lambda_T}l\sqrt{\frac{1}{2}\xi}\left[\Phi\,(\xi,\cos\theta)-2\left(1+\xi\cos\theta\right)\cos\theta+\xi\right]^{\frac{1}{2}}sign\theta \qquad (22)$$

where $\xi=\frac{\rho}{l}$ and

$$\Phi(\xi,\cos\theta)=\sqrt{\xi^2+4\xi\cos\theta+4} \qquad (23)$$

Function $\Phi(\xi,\cos\theta)$ is related to the generating function of the Legendre polynomials, and thus can be represented by these polynomials

$$\Phi(\xi,\cos\theta)=2+\xi\cos\theta+\sum_{n=2}^{\infty}(-1)^n\frac{A_n\left(\cos\theta\right)}{2^{n-1}\left(2n-1\right)}\xi^n \qquad (24)$$

where

$$A_n(\cos\theta)=P_{n-2}(\cos\theta)-P_n(\cos\theta) \qquad (25)$$

a Legendre polynomial in a trigonometric representation. As a result, the temperature distribution in the neighborhood of the right vertex of the crack can be written as

$$T(\xi,\cos\theta)=T_0+\frac{q_T}{\lambda_T}l\sqrt{\frac{1}{2}\xi}\,[4\sin^2\theta-\xi(2\cos^2\theta-\cos\theta-1)+$$
$$+\sum_{n=2}^{\infty}(-1)^n\frac{A_n(\cos\theta)}{2^{n-1}(2n-1)}\xi^n\,]^{\frac{1}{2}}\,sign\,\theta \qquad (26)$$

This formula has the advantage of formula (22), since it allows easily to build various approximations on the parameter $\xi=\frac{\rho}{l}$, ie, to investigate the temperature field at different distances from the top of the crack. In particular, to limit members of the second order ξ, after some transformations will get

$$T(\xi,\theta)=T_0+\frac{q_T}{\lambda_T}l\sqrt{2\xi}\left[\sin\frac{\theta}{2}+\frac{1}{4}\xi\sin\frac{3\theta}{2}-\frac{1}{32}\xi^2\sin\frac{5\theta}{2}\right] \qquad (27)$$

This formula and will use in the further analysis.

We determine the magnitude of the gradient of the temperature field in the vicinity of crack tip.

$$grad\,T=\frac{\partial T}{\partial x}i+\frac{\partial T}{\partial y}j \qquad (28)$$

Differentiating the formula (27), we obtain

$$a)\ \frac{\partial T}{\partial x}=-\frac{q_T}{\lambda_T}\frac{1}{2\sqrt{\xi}}\left[\left(1-\frac{3}{4}\xi\right)\sin\frac{\theta}{2}+\frac{5}{32}\xi^2\sin\frac{3\theta}{2}\right] \tag{29}$$

$$б)\ \frac{\partial T}{\partial y}=\frac{q_T}{\lambda_T}\frac{1}{2\sqrt{\xi}}\left[\left(1+\frac{3}{4}\xi\right)\cos\frac{\theta}{2}-\frac{5}{32}\xi^2\cos\frac{3\theta}{2}\right]$$

Hence, the gradient magnitude $|grad\,T|$ with accuracy to terms of second order at ξ equal

$$|grad\,T|=\frac{q_T}{2\lambda_T}\left[\sqrt{\frac{\rho}{l}}+\frac{3}{4}\cos\theta+\frac{7}{16}\left(\frac{\rho}{l}\right)^{\frac{3}{2}}\left(1-\frac{19}{24}\cos^2\theta\right)\right] \tag{30}$$

In particular, in the vicinity of the vertex is

$$|grad\,T|=\frac{q_T}{2\lambda_T}\sqrt{\frac{l}{\rho}} \tag{31}$$

and the heat flux density will be equal to

$$|q|=\frac{q_T}{2}\sqrt{\frac{l}{\rho}} \tag{32}$$

From these formulas it is seen that in the vicinity of crack tip the magnitude of the heat flux and temperature gradient does not depend on the polar angle θ and reaches very large values. This confirms the picture in Fig. 7. Large values of temperature gradient near the top of the crack cause a large thermal stress in the volume surrounding the vertex.

It is known that in the mechanical field crack is the hub voltage: local voltage near the peaks greatly exceed the stress away from her. Now, on the basis of obtained results it can be concluded that the temperature field of the crack is a hub of heat flow: the heat flow and the gradient of the temperature field near cracks is much more than away from her.

Thus, the analysis of the temperature solution of the boundary value problem shows that the crack distorts the temperature field characteristic of the sample without cracks. This distortion is localized near the cracks, and distortion is determined by the size of the crack. Hence the important practical conclusion: possible thermal diagnostics to detect internal defects. This may be one of the efficient NDT techniques.

3. The temperature field at the crack tip

Calculate the temperature near the top of the crack, i.e. where concentrated heat flux. The real crack is a crack with asymptotically converging shores. The distance between the banks (the crack opening) gradually decreases down to interparticle distances (interatomic and intermolecular). It follows that at the end of the crack is the area where the significant strength of interatomic and intermolecular interactions of the crack faces, and to neglect them in any way is impossible. This is the basic theory of brittle cracks Barenblatt (1964) and δ_k the theory of Leonov-Panasyuk (1968).

Interparticle adhesion force near the fracture tip ensures smooth closure of its banks and the finiteness of stresses near the top. These forces play a significant role in the narrow portion near the edge of the crack, where the crack opening does not exceed the range of the adhesion forces. The shape of the cracks in this area is determined by the nature of the adhesion forces and does not depend on crack length and external load. The rest of the shape of the cracks depends on the external stresses of mechanical or thermal origin.

The tip of the crack is a small "beak" (figure 9), the stress state and the dimensions of which are entirely determined by the interatomic forces and intermolecular interactions and do not depend on external loads and temperature. In other words, the beak cracked Autonomous in relation to the entire crack, and with the growth of the latest moves ahead of her, without changing neither its size nor shape [12, 13].

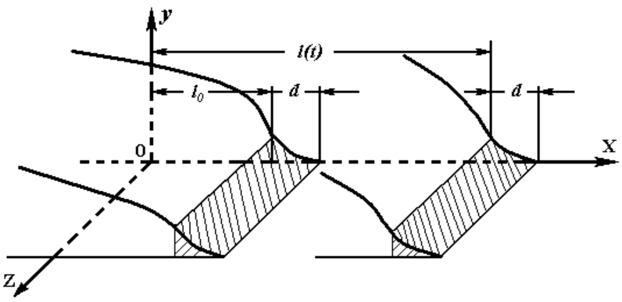

Fig. 9. The beak cracked

In the monograph (Landau and Livshits, 1978) gives an estimate of the order of magnitude of the size of the areas where significant forces of interparticle adhesion: if the d length of this zone $d \approx \left(a r_0^2\right)^{\frac{1}{3}}$, where a is the atomic size, r_0 the range of interparticle (intermolecular) forces of adhesion, i.e. d, although large compared with atomic dimensions a, at the same time small compared to the intermolecular distance r_0. For the numerical evaluation as a value a will accept the atomic radius of the carbon atom, i.e., the radius of the outer orbitals of this atom. There is given the radii of atoms of different elements, in particular, for carbon we take $a = 0.620 \cdot 10^{-10}$ м(Godovikov, 1979). Radius of action of intermolecular forces of adhesion can take is equal to the average intermolecular distance, in particular, for PMMA it is $r_0 = \lambda = 12 \cdot 10^{-4}$ мкм $= 12 \cdot 10^{-10}$ м. Then get in order of magnitude $d \approx 4.5 \cdot 10^{-10}$ м, i.e. d large compared to atomic dimensions a, but small compared to the intermolecular distance r_0. This is understandable, because at distances larger intermolecular bonds between the atoms already almost do not work and the crack does not interact. As the ultimate upper limit of the length of the "beak" you can take the intermolecular distance. So take as an estimate that the size of the "beak" is $d \approx (4.5 - 12) \cdot 10^{-10}$ м.

In δ_k theory Leonov-Panasyuk [13] the section of the crack near its top, i.e. the beak is cracked where significant traction is called the zone of weak ties. In the area of interparticle when stretched, weakened, but still partially broken. Using the results of this theory, we are able to describe in more detail the beak cracked (its size, shape, and opening cracks in beak). In particular, the limit value of the size of the "beak" get equal.

$$d = \frac{\pi \lambda^2 E}{16\left(1 - \nu^2\right)\alpha_n} \qquad (33)$$

where λ is the average intermolecular distance, which defines the maximum opening of the "beak", α_n is the specific surface energy of destruction, E is young's modulus, ν is Poisson's ratio. From this formula clearly demonstrates the independence of the size of the "beak" from external loads. To evaluate the size of the "beak" will take the data for PMMA: $\lambda = 1.2 \cdot 10^{-9}$ м, $E = 3.93 \cdot 10^9 \frac{н}{м^2}$, $\nu = 0.25$, $\alpha_n = 0.15 \frac{дж}{м^2}$, [11,13]. Obtained, $d = 7.9 \cdot 10^{-10}$ м i.e. almost the same value as above. Compare these results with the diameter fluctuation volume, which according to thermalfluctuation failure theories [11] there is an elementary acts of destruction. For PMMA [11] gives values $\frac{1}{8}\left(\frac{d}{l}\right)^{\frac{3}{2}} \approx 0.005$, hence the diameter fluctuation amount is $d \approx \left(2V_a\right)^{\frac{2}{3}} \approx (4.6 - 6.5) \cdot 10^{-10}$ м, i.e., the diameter fluctuation of the volume and the length of the "beak" the cracks are almost identical. This coincidence implies that "beak" is microobject near the cracks (fluctuation amount) which draws the basic acts of destruction. More specifically, the microvolume happen the final acts of "doriane" strained relations. The primary acts of pre-destruction happen in front of the crack in particular arise where the conditions, where large stresses develop inelastic phenomena.

In a simplified model of the crack, when it is represented by a mathematical cut elastic plane has no thickness, the temperature profile on the crack and out of it has the appearance as shown in figure 2. The area of the cracked-section, where the heat does not pass through its banks, abruptly gives way to the area of heat conduction where the material retains continuity. The real crack is a crack with asymptotically converging shores [11]. In the Central part of the crack do not interact and are separated by a layer of air (or even vacuum) with a low conductivity. The thermal resistance is very large, and the heat through the Bank fails. Therefore, the upper shore is heated, and the bottom remains cold. In "the beak" crack, interparticle connection, partially broken, although stretched and weakened. It's warm "seeps" through the crack in these relations and, because of this there is a smooth transition from the region of the bow heat to full conduction. Therefore, the temperature profile at the upper end and gradually decreases to an average level. The thermal resistance in the "beak", although large, but of course. Therefore, the "beak" is also heated, although less than the Central portion of the upper shore. In the same field far from the crack where the continuity is not broken, the thermal resistance is even less (almost zero), so there remains the average temperature. Thus, the "beak" crack "overheated" in comparison with the average level. To find this overheating, refer to the formula (27) describing the temperature distribution near the top of the crack. Averaging the amount of "beak" from the top of the shore get

$$T_* = T_0 + \frac{q_T l}{\lambda_T}\left(\left(\frac{d}{l}\right)^{\frac{1}{2}} - \frac{1}{8}\left(\frac{d}{l}\right)^{\frac{3}{2}}\right) \qquad (34)$$

The beak is about one-tenth of the crack length, ie $\frac{d}{l} \approx 0.1$. Then $\left(\frac{d}{l}\right)^{\frac{1}{2}} \approx 0.35$, and $\frac{1}{8}\left(\frac{d}{l}\right)^{\frac{3}{2}} \approx 0.005$. Therefore, the second term can be neglected, and get

$$T_* = T_0 + \frac{q_T}{\lambda_T}\sqrt{l\,d} \qquad (35)$$

Hence, overheating of the "beak" of the upper Bank equal

$$\Delta T = \frac{q_T}{\lambda_T}\sqrt{l\,d} \qquad (36)$$

In "beak" temperature of the upper and lower banks is the same. Therefore, this expression must be doubled. Finally overheating of the "beak" is equal to

$$\Delta T = \frac{2q_T}{\lambda_T}\sqrt{l\,d} \qquad (37)$$

It is seen that the overheating of the "beak" depends on the size of the crack, increasing as $\sqrt{l}$. On the fluctuation stage of development of the crack increases in size by 6-10 times. Therefore, overheating of the "beak" increases as the crack growth in the 2.4-3.2 times.

Compare the temperature of the "beak" with a maximum temperature of the top Bank in the middle. From the formula (18), we obtain

$$T_{берег} = T_0 + \frac{q_T}{\lambda_T} l \qquad (38)$$

Comparing with (37), we obtain

$$\frac{\Delta T_{берег}}{\Delta T} = \frac{1}{2}\sqrt{\frac{l}{d}} \approx 1.44 \qquad (39)$$

i.e. the maximum temperature on the upper shore is almost one and a half times greater than the temperature of the "beak". This is enough to provide a constant flow of heat into the beak at the growth of cracks.

In conclusion, express the overheating of the "beak" using the parameters defining its size in the formula (33).

Get

$$\Delta T = \frac{2q_T \lambda}{\lambda_T} \sqrt{\frac{\pi l E}{\left(1-\nu^2\right)\alpha_n}} \qquad (40)$$

Thus, if under mechanical loading in the beak cracks, there is a constant flux of elastic energy during thermal loading – the flow of thermal energy. He and the other contributes to thermal fluctuation elementary acts of destruction and a crack develops, gradually accelerating until the complete destruction

4. Conclusions

1. When exposed to a steady heat flow in a sample with a crack is a significant increase in thermal stress caused by a local increase of the temperature gradient in the vicinity of the crack.
2. The crack grows under the action of local stresses near its summit. It doesn't matter what factor created by these local stresses.
3. The crack distorts the temperature field characteristic of the sample without cracks. This distortion is localized near the cracks, and distortion is determined by the size of the crack. On the crack in addition to the displacement jump occurs, the jump in temperature proportional to the power of the external heat flux and size of the crack. In the mechanical field, a crack is a hub (local amplifier) voltage, and temperature field, in addition to the hub of the heat flow.
4. The real crack is a crack with asymptotically converging shores. The consequence of this is the existence of a "beak" cracks, i.e., the plot at the end of the crack, where the significant strength of interparticle coupling its shores. These forces provide a smooth closure of the crack faces and limb, stress, as well as a limb component of heat flow near its summit. The beak is Autonomous in relation to the crack, and when movement of the latter moves with it, without changing any size or shape.
5. "Thermal resistance" of the beak cracked more significantly than far from it. As a result, the bill is "overheated" in comparison with the average level. Numerical evaluation for PMMA showed that depending on the initial sizes of cracks overheating of the beak is from a few tenths to a few degrees, and when the cracks, the value of overheating of the beak varies in 2.5 – 3 times.
6. The existence of the mechanical equivalent of heat flux, i.e. the equivalent mechanical stress, which is equivalent to the action of heat flow.

Bibliographic references

Barenblatt, G. I. (1964). On some of the General concepts of the mathematical theory of brittle cracks. *Applied mathematics and mechanics*, *28*(4), 630-643.

Bartenev, G. M. (1984). Strength and fracture mechanism of polymers. M.: Chemistry, 280 p.

Dimitrienko Yu. I., Sokolov, A. P. (2013). The Study of processes of destruction of composite materials on the basis of the method of asymptotic homogenization. *Engineering journal: science and innovations, 1*.

Dimitrienko, Y. I., Sokolov, A. P. (2010). Elastic properties of composite materials. *Mathematical Models and Computer Simulations*, *2*(1), 116-130.

Dimitrienko, Yu. I. (1995). Thermal stresses and heat mass-transfer in ablating composite materials// Int. Journal of Heat Mass Transfer.- 1995.- Vol.38.- No. 1. P. 139-146.

Dimitrienko, Yu. I., Sokolov, A. P. (2012). Multiscale modeling of elastic composite materials. *Mathematical modeling, 24*(5), 3-20.

Dimitrienko, Yu.I. (1997). Thermal Stresses in Ablative Composite Thin-Walled Structures under Intensive Heat Flows. *Int. Journal of Engineering Science, 35*(1), 15-31.

Finkel, V. M. (1977). *Physical basis of inhibition of destruction*. M.: Metallurgy, 359 p.

Godovikov, A. A. (1979). *Crystal chemistry of simple compounds*. Novosibirsk: Science, 147 p.

Kartashov, E. M. (1985). *Analytical methods in the theory of thermal conductivity of solids*. M.: Higher school, 480 p.

Kartashov, E. M. (1991). *Modern concepts of the thermal fluctuation kinetic theory of strength of polymers. Results of science*. Moscow: Nauka, T. 27, pp. 2-48.

Landau, L. D., Livshits, E. M. (1978). *Theory of elasticity*. Moscow: Nauka, 358 p.

Panasyuk, V. V. (1968). *Limiting equilibrium of brittle bodies with cracks*. Kiev: Naukova Dumka, 246 p.

Regel, V. R., Slutsker, A. I., Tomashevsky, E. E. (1974). *Kinetic nature of strength of solids*. Moscow: Nauka, 560 p.

Si, G. S. (1963). On the singular character of thermal stresses near the top of the crack. Applied mechanics. *Proceedings of the American society of mechanical engineers (Russian translation), 29*(3), 157-159.

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